Radiation from Planar Resonators

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Abstract **—Radiation from planar resonators is troublesome because it tends to inject spurious signals into neighboring circuits. Power radiated from microstrip-based resonators is calculated by integration of a Green's function with assumed currents, a method that is convenient and is thought to be more accurate than methods used in earlier calculations of radiation Q. Resonators consisting of two coupled microstrips excited in the odd mode are found to radiate very much less than conventional single microstrip resonators or U-shaped "hairpin" resonators suggested earlier. However, when the resonator is loaded by a semiconductor device, as in an oscillator, radiation is increased. Asymmetries in these resonators, arising from output coupling or fabrication errors, introduce even-symmetric currents which radiate much more strongly than odd-mode currents; the effects of such asymmetries on radiated power are estimated. On the basis of these findings a convenient geometry for high-power planar oscillators with low radiation is proposed.**

I. Introduction

RESONANT circuits may be expected to be useful in planar microwave technology. One of the more important applications is in planar oscillators, which are used as local oscillators in receivers and as radiation sources in radar transmitters. In such cases the power levels are high, and if the resonator tends to radiate, undesirably large spurious signals may be injected into neighboring circuits. A secondary consideration is actual power loss from the resonator with attendant reduction of total Q. Radiation is not usually the dominant loss mechanism, but it tends to be especially large in planar resonators for use at millimeter-wave frequencies, because radiation increases rapidly with the ratio of substrate thickness to free-space wavelength. Because of practical limitations on substrate thickness, that ratio tends to be larger at high frequencies.

In this paper we shall consider radiation losses from microstrip resonators, using a Green's function method for calculating power radiated into space modes and surface modes. After verifying the technique by comparisons with earlier calculations and our own experiments for the case of conventional straight half-wave resonators, we go on to consider modified resonators with odd symmetry, from which radiation loss is greatly reduced. In practical oscillators these resonators will be loaded by the active device; this loading is found to significantly affect the radiated power. It may happen that resonators intended to be symmetrical actually have some asymmetry, either because of fabrication errors or because of output coupling; we estimate the increase in radiation to be expected in such cases. Finally we conclude with some general design suggestions for oscillators with low radiation loss.

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II. Calculation of Radiation Loss

The earliest estimates of radiation loss from microstrip resonators were based on approximate methods, the accuracies of which were difficult to estimate [1]–[3]. A greatly improved calculation was supplied by Van der Pauw [4]. Although nearly free of *ad hoc* assumptions, the method of Van der Pauw did involve the assumption of a thin substrate, which again was hard to quantify. All of those methods, as well as the one used in this paper, involve, for simplicity, an assumed current distribution. It is also possible to calculate the current distribution in detail, and the power radiated by a microstrip stub has recently been found in this way by Harokopus and Katehi [5]. A hazard in much of the earlier work is the idea that the radiation is from the open end of the resonator, or, in general, from just one part of a circuit. Actually the entire circuit radiates together, with interference between radiation from one part and that from another. Thus the only safe approach is to calculate the fields radiated from each point in the entire circuit, and then add these fields vectorially. The calculation is simplified in the case of resonators, because they are usually coupled only weakly to the remainder of the circuit. The currents in the resonator are generally much larger than those outside it; thus the contribution to the total radiation from the remainder of the circuit can reasonably be neglected.

Our approach to finding the radiation from an arbitrary collection of microstrips involves finding the Green's functions for radiation into space and into surface waves, and integrating these over appropriate angles to obtain the total radiation. The Green's functions used are those of a current element located on the surface of a dielectric slab backed by a ground plane, with the direction of current flow parallel to the surface. Since we are only interested in the distant radiation fields, it is convenient to obtain these functions by the reciprocity method [6], [7]. Let the polar axis be normal to the surface and let the $\phi = 0$ direction coincide with the direction of current flow. The Green's functions for space radiation with electric field perpendicular to the plane of incidence is then found to be

$$E_\phi = \frac{jI_0 l\omega\mu}{2\pi}\left[\frac{\tan u \sin\phi\cos\theta}{\cos\theta\tan u - j\left(\dfrac{u}{kh}\right)}\right]\left(\frac{e^{-jkr}}{r}\right) \tag{1}$$

and that for electric field in the plane of incidence is

$$E_\theta = \frac{jI_0 l\omega\mu}{2\pi}\left[\frac{\tan u\cos\phi\cos\theta}{\tan u - jn^2\left(\dfrac{kh}{u}\right)\cos\theta}\right]\left(\frac{e^{-jkr}}{r}\right) \tag{2}$$

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where I_0 and l are the magnitude and length of the current element, n and h are the refractive index and thickness of the dielectric slab, $k = \omega / c$, and

$$u = kh\sqrt{n^2 - \sin^2\theta}.$$

We assume that because of the small thickness of the dielectric, all surface modes except TM_0 are cut off. The magnetic field radiated into this mode (measured at the ground plane) is

$$H_\phi = \frac{I_0 l k_f}{h_e(2\pi\beta)^{1/2}} \sin(k_f h)\cos\phi\left(\frac{e^{-j\beta r}}{\sqrt{r}}\right) \qquad (3)$$

where, following Kogelnik [8], k_f is the root of the equation

$$(k_f h)\tan(k_f h) = n^2\sqrt{(n^2-1)(kh)^2-(k_f h)^2}$$

and $\beta^2 = n^2k^2 - k_f^2$. The power radiated in the direction ϕ per unit angle is

$$S(\phi) = \frac{(I_0 l)^2 \eta_0 k_f^2 \sin^2(k_f h)\cos^2\phi}{8\pi k h_{\text{eff}} n_f^2} \qquad (4)$$

where $\eta_0 = (\mu_0/\epsilon_0)^{1/2}$, $n_f^2 = (k_f^2 + \beta^2)/k^2$, and

$$h_{\text{eff}} = h + \frac{1}{\left[(n_f^2-1)k^2 - k_f^2\right]^{1/2}} \frac{n_f^2 k^2}{\beta^2(n_f^2+1) - n_f^2 k^2}.$$

The radiation Q of some open-ended half-wave linear microstrip resonators has been calculated using (1)–(4). The assumed current distribution is a line current located at the center of the strip, with amplitude proportional to $\sin(\epsilon_{\text{eff}}/\epsilon_0)^{1/2}kz$, where ϵ_{eff} is the well-known effective permittivity of the microstrip. Fig. 1 shows the results for the case $\epsilon_R = 12$, $w/h = 1$ (where w is the width of the microstrip, ϵ_R is the relative permittivity of the dielectric, and h is its thickness) as a function of the normalized dielectric thickness kh. Also shown for comparison are values obtained by performing the integrals of [4, eq. (26)], as well as our own experimental measurements. The latter were made using Emerson & Cumings Stycast Hi-K material at frequencies between 0.2 and 2.0 GHz by the usual gap-coupled reflection method [9]. A removable aluminum cover was used to separate radiation loss from the other losses [10]. The experiments agree well with (1)–(4). The theory of Van der Pauw also agrees well for thinner substrates, but diverges increasingly for $(kh) > 0.075$.

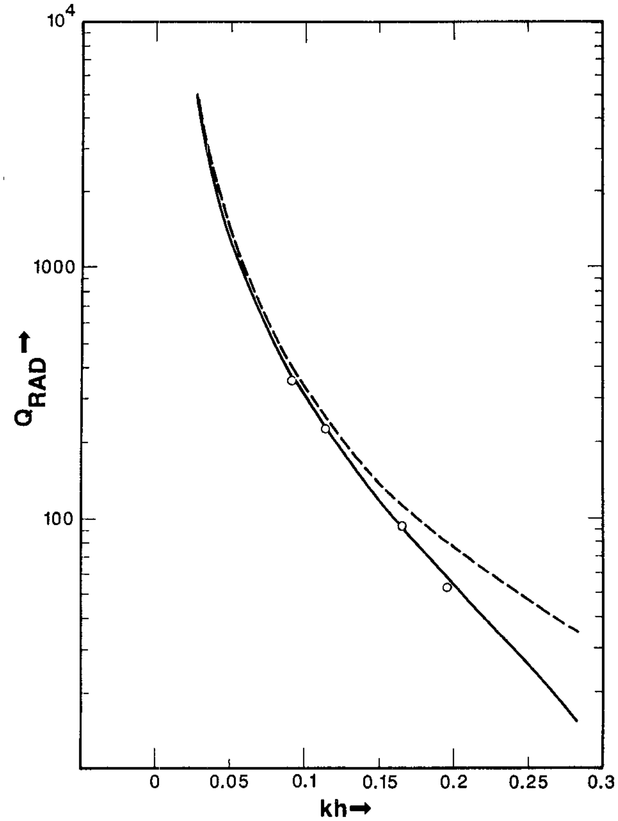

Fig. 1. Radiation Q of open-ended half-wave linear microstrip resonators with $\epsilon_R = 12$, $w/h = 1$ as a function of normalized substrate thickness. Solid curve and experimental points from this work; dashed curve, ref. [4].

III. Resonators with Reduced Radiation Loss

The U-shaped "hairpin" configuration has been suggested [11]–[13] as a way to reduce the power radiated by microstrip resonators. The improvement arises because the two parallel sides of the "U" function as though they were two parallel coupled microstrips excited in the odd mode. The currents in these two microstrips are equal and opposite, resulting in cancellation of their radiation fields. One expects this cancellation to reduce radiated power by roughly a factor of $(kd)^2$, where d is the center-to-center spacing of the strips. A shortcoming of this sort of resonator is that the radiation of the bottom, or curved, part of the "U" is not canceled; in fact we observe that by far the greatest part of the radiated power comes from this part of the resonator. Resonators with higher radiation Q can be obtained by eliminating this cross-connection, and using a configuration consisting simply of two parallel edge-coupled microstrips of equal length. Fig. 2 shows calculated radiation Q for open-ended half-wave resonators of this kind with $\epsilon_R = 12$ and $w/h = 1$ for both strips, as a function of s/w (where s is the distance between the inner edges of the strips.[1] The radiation Q's predicted in this figure are enormously large, ranging from 20000 for the thickest dielectrics to more than 10^7 for the thinnest ones. Of course overall Q's of this order cannot be achieved in practice because of ohmic and dielectric losses. In fact, the total Q of the dual resonator may be lower than that of a conventional single resonator, because ohmic loss is increased as a consequence of current crowding at the inner edges. However the results are still of interest because they imply a greatly reduced degree of radiation coupling between the resonator and other circuits. In experiments we have observed radiation Q's for this type of resonator on the order of 7000, even with $(kh) > 0.1$.

IV. Resonators for Planar Oscillators

In the case of oscillators the active device becomes part of the resonant circuit, and its effects must be considered. A convenient geometry for a planar oscillator with reduced

[1] In performing this calculation, the strips were modeled as two parallel line currents spaced by a distance d. The value of d was taken to be the root-mean-square average separation of the actual current distribution, as determined by an electrostatic calculation. Actually, the value of d obtained in this way does not differ much from the value obtained by simply taking the spacing between the centers of the strips.

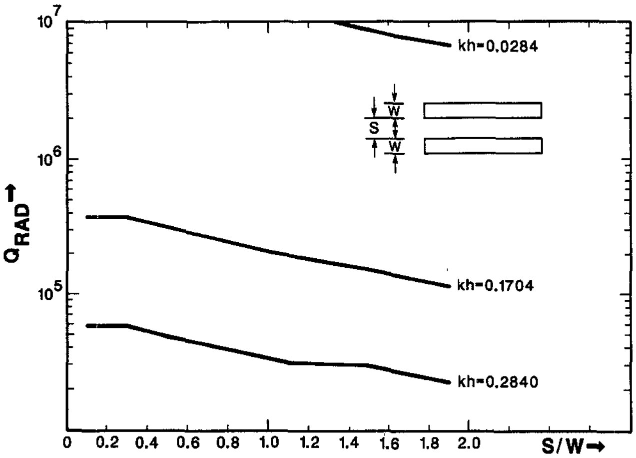

Fig. 2. Calculated radiation Q of resonators consisting of two parallel open-ended microstrips with $w/h=1$, $\epsilon_R=12$, as function of spacing, for three values of normalized substrate thickness.

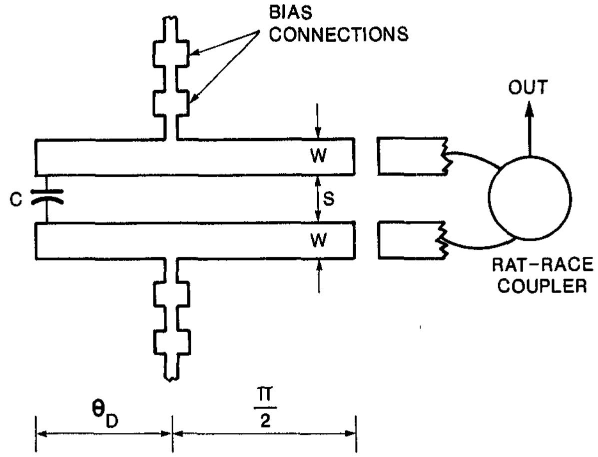

Fig. 3. Model of an oscillator composed of a parallel-microstrip resonator and semiconductor device.

radiation is shown in Fig. 3. Here the active device is modeled by its capacitance C. A rat-race coupler is used to combine the two gap-coupled outputs in such a way as to avoid disturbing the symmetry. At a position approximately 1/4 wavelength from the output gaps a voltage minimum occurs, providing a convenient point for bias connections. The angular distance θ_D is determined by $\cot\theta_D = Z_{0o}\omega C$, where Z_{0o} is the characteristic impedance of the odd mode of the coupled microstrips. Radiation Q's for this type of resonator are shown in Fig. 4. The effect of the unbalanced cross-current through C is easily seen. This current is largest when θ_D is near an integer multiple of π, a condition which unfortunately is likely to arise whenever a large, high-power active device is used. The radiation Q that occurs with $\theta_D \approx \pi$ is actually about twice as large as that with $\theta_D \approx 0$, although this does not imply that radiated power is less, since the energy stored in the longer resonator will be larger. If space is available one might prefer this longer configuration in order to ensure a well-proportioned resonator.

V. Unbalanced Resonators

The rat-race coupler used in Fig. 3 may be undesirable because it will consume a great deal of space. One might instead use only a single gap-coupled output and attempt to balance the resonator by making one arm slightly longer than the other. It may not be possible to perform this compensation with perfect accuracy; moreover, asymmetries may creep into the resonator from other causes. If this should happen, the resonant modes of the resonator will no longer be purely even or odd, but mixtures of the two. Since current distributions with even symmetry radiate much more strongly than those of odd symmetry, one expects that even a small admixture of the even distribution will tend to reduce the radiation Q.

In order to observe the significance of this effect, let us consider the unbalanced resonator shown in Fig. 5. One of four ports of an open-ended half-wave dual resonator is loaded with a load susceptance B_L. The currents in the two lines, $i_1(z)$ and $i_2(z)$, are composed of the even- and odd-mode currents i_e and i_o according to $i_1 = i_e + i_o, i_2 = i_e - i_o$. In this case $i_e = I_e \sin k_e z$ and $i_o = I_o \sin k_o z$, where I_e and I_o are the even- and odd-mode amplitudes and k_e and k_o

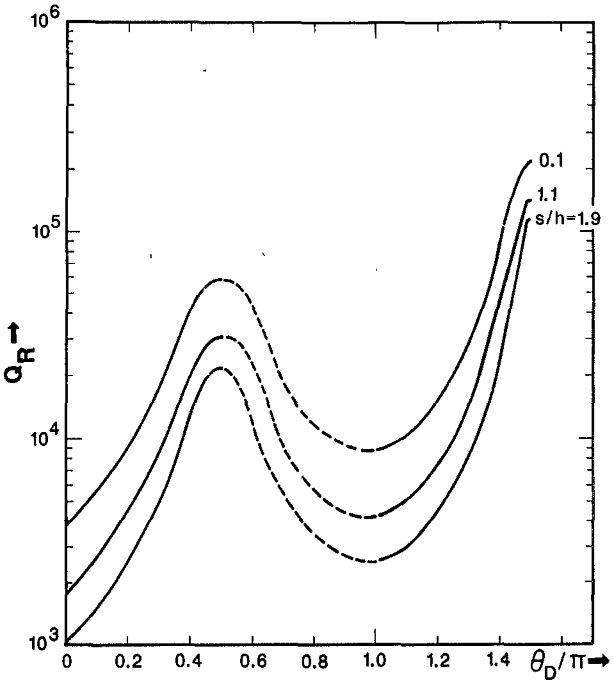

Fig. 4. Radiation Q for resonators of the type shown in Fig. 3, with $w/h=1$, $kh=0.284$, $\epsilon_R=12$. The angular length θ_D is determined by the capacitance C. For values of θ_D between $\pi/2$ and π (dashed lines) an inductance, rather than a capacitance, is required.

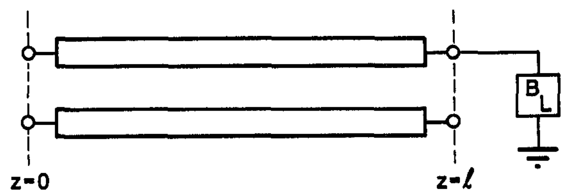

Fig. 5. Unbalanced parallel-line resonator.

are the even- and odd-mode propagation constants. We apply the boundary conditions $i_1(z=0)=i_2(z=0)=i_1(z=l)=0$ and $i_2(z=l)=jB_L\,v_2(z=l)$, and obtain the characteristic equation

$$[Z_{0o}\cot k_o l + Z_{0e}\cot k_e l]B_L = -2. \tag{5}$$

Let us define $\phi_o \equiv k_o l$, $K \equiv k_e/k_o$, and let $x \equiv I_e/I_o$ be the ratio of even to odd amplitudes present in the resonant mode. When $B_L=0$, odd-mode resonance is obtained at $\phi_o=\pi$. When B_L is introduced, ϕ_o changes to $\pi+\delta$ and the resonant frequency of the oddlike mode increases to $(1+\delta/\pi)$ times its unperturbed value. An approximate solution for (5), valid for $k\delta \ll \pi/2$, can be found by expanding the trigonometric functions to first order in δ. Thus, for example, we set

$$\cot k_e l = \cot K\phi_o \approx \frac{1-\delta\tan(k\pi)}{\tan(k\pi)+\delta}.$$

On discarding quadratic and higher powers of δ we obtain the approximate result for the frequency pulling of the lowest odd mode,

$$\delta \approx -\frac{\tan(k\pi)}{\left(\dfrac{2\tan k\pi}{B_L Z_{0o}}+K+C\right)} \tag{6}$$

where $C \equiv Z_{0e}/Z_{0o}$. The fractional mode mixing $x \equiv I_e/I_o$ is found from the boundary condition $i_1(z=l)=0$, from which we have

$$I_e\sin k_e l + I_o \sin k_o l = 0$$

$$x = -\frac{\sin k_o l}{\sin k_e l} = -\frac{\sin\phi_o}{\sin K\phi_o}. \tag{7}$$

Setting $\phi_o=\pi+\delta$, retaining only first-order terms in δ, and taking ϕ from (6), we obtain

$$x \approx -\left[\frac{2\sin(K\pi)}{B_L Z_{0o}}+(K+C)\cos(K\pi)\right]^{-1}. \tag{8}$$

Equation (5) can also be solved numerically. Some results for the case of $\epsilon_R=12$, $w/h=1$, $s/h=0.1$, $kh=0.17$ ($K=1.38$, $Z_{0e}/Z_{0o}=2.42$) are shown in Fig. 6. We observe that as the product $B_L Z_{0o}$ increases, the resonant frequency of the lowest-order oddlike mode slowly decreases, and it increasingly acquires an even-mode current component, as indicated by increasing x.

We can estimate the effective radiation Q of the unbalanced resonator in the following way. Let us assume that the current distribution in each of the two parallel microstrips is not greatly different from what it would be if the two strips were far apart and B_L were zero. Let the energy stored in a single half-wave microstrip resonator with maximum current I be called $U(I)$. For an ideal, balanced odd resonator, with odd-mode current I_o defined as above, the stored energy would then be $2U(I_o)$. For a resonator that is only slightly unbalanced, the total stored energy is nearly the same as the energy stored in the odd mode. However, because of the even mode's much greater propensity to radiate, the total radiated power is likely to be dominated by that of the even current component; let us call this radiation P_{even}. Thus the effective radiation Q of the unbalanced resonator is given

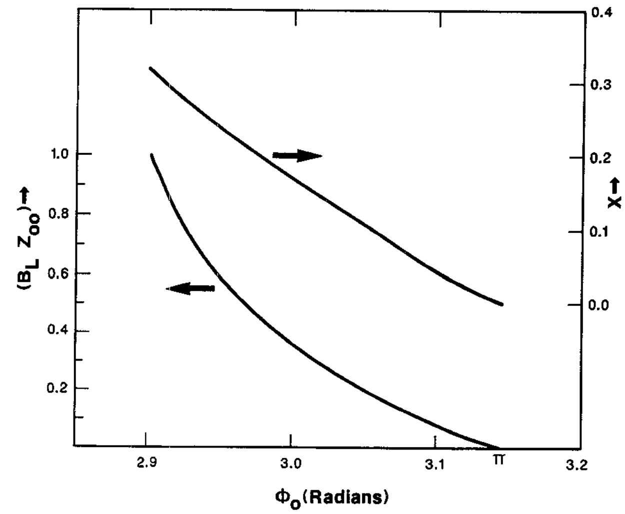

Fig. 6. Resonant frequency and fractional even-mode excitation (right scale) for lowest-order oddlike mode of unbalanced resonator, as functions of normalized load susceptance (left scale). These data are for $\epsilon_R=12$, $w/h=1$, $s/h=0.1$, $kh=0.17$.

approximately by

$$Q_{\text{effective}} \approx \frac{2\omega U(I_o)}{P_{\text{even}}}. \tag{9}$$

To find P_{even} we can make use of the known radiation of single half-wave microstrip resonators. The power radiated by such a resonator, on which the maximum current is I, is

$$P_{\text{single}} = \frac{\omega U(I)}{Q_{\text{single}}} \tag{10}$$

where Q_{single} is the radiation Q of a single half-wave microstrip resonator. Now for the double resonator, each strip carries an even-mode current $I_e = xI_o$. These two currents radiate in phase, so the total radiated power is given by

$$P_{\text{even}} = 4\frac{\omega U(xI_o)}{Q_{\text{single}}} = \frac{4x^2\omega U(I_o)}{Q_{\text{single}}}. \tag{11}$$

Combining (9) and (11) we have

$$Q_{\text{effective}} \approx \frac{Q_{\text{single}}}{2x^2}. \tag{12}$$

The effect of asymmetry will become important whenever $Q_{\text{effective}}$ is smaller than Q_R for the pure odd mode. Using the data of Fig. 1, we estimate that an asymmetry giving $x=0.089$ would reduce the radiation Q of a dual resonator with $kh=0.17$ from the enormous values predicted in Fig. 3 to the order of 5000.

VI. Design Suggestions for Oscillators

It seems desirable to construct planar oscillators, especially high-power ones, in such a way that radiation is minimized. This will reduce coupling of the large oscillator signal into other parts of the circuit. A particularly favorable configuration appears to be the one shown in Fig. 7. Here a second, vertical ground plane is used at one side of the circuit. Its function is to act as a mirror plane, so that only half of the U-shaped resonator needs to be fabricated. This is advantageous because the resulting dual resonator, consisting of the single microstrip and its image, is automatically

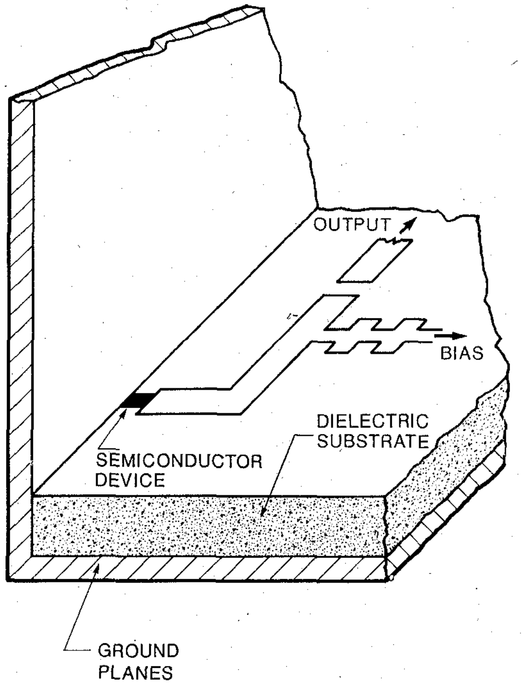

Fig. 7. Proposed configuration for a planar oscillator with low radiation loss.

symmetrized, so that radiation is minimized. This will be the case even with single gap coupling of the output, which is very convenient. In addition, the vertical ground plane should be helpful in heat-sinking the semiconductor device.

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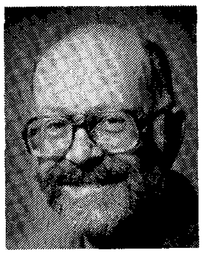

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